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D0-Branes, Constrained Instantons and D=4 Super Yang-Mills Theories

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Abstract: We consider in more detail the role of D0-branes as instantons in the construction of $SU(N)$ Super Yang-Mills and Super QCD theories in four space-time dimensions with D4, D6 and NS-branes. In particular, we show how the D0-branes describe both the exact and constrained instantons and reproduce the correct pattern of lifting of zero modes on the various branches of these models.

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1. Introduction

In this paper we consider the by now standard construction of four-dimensional super Yang-Mills (SYM) theories with Dirichlet branes (D-branes) [1,2] (see also [3,4,5,6,7,8,9,10]) and discuss how the world-line dynamics of D0-branes bound to D4-branes describes field theory SYM instantons (a more geometrical approach was developed in refs. [11]).

The basic construction of SYM instantons on a Dp -brane world-volume, is as bound states with $D(p-4)$ -branes [12,13,14,15]. In the case of the construction of ref. [2], the $SU(n_c)$ SYM theory is described by n_c D4-branes, so we must consider D0-branes bound to D4-branes. The five-dimensional world-volume of the D4-branes is then compactified to the four-dimensional physical space and, to make a bound D0-brane describe an instanton in the four dimensional physical space, the D0-brane euclidean-time world-line must lie along the compactified direction of the D4-brane. Although the details of such reduction depend on the amount of supersymmetry ($N = 4, 2, 1, 0$) that we want to preserve in space-time, the basic properties of the instanton moduli space only depend on the geometry of D0-D4 branes bound states, which we review in this introduction.

The collective dynamics of the instantons is specified by a point action and an appropriate measure over instanton moduli space. In the present D0-branes construction, both ingredients appear as the dimensional reduction down to zero dimensions of the D0-brane one-dimensional collective dynamics. In particular, the instanton moduli space as a hyper-Kahler quotient appears as a Higgs branch of the world-line theory of the D0-branes. The basic terms in the world-line action are the static Born-Infeld action of the D0-brane

$$S_{BI} = \tau_0 \int \sqrt{\det h_{\text{induced}}} \longrightarrow M_{D0} \int d\tau, \quad (1.1)$$

and a source term coming from the Chern-Simons couplings on the D4-branes world-volume

$$S_{\text{source}} = i \frac{\mu_4}{2} \int_{\Sigma_{4+1}} A_{D0}^{RR} (2\pi\alpha')^2 F \wedge F, \quad (1.2)$$

which relates the RR photon of the type-IIA string theory to the theta parameter of the four-dimensional instanton.¹ There are also non-trivial gauge interactions specifying the

¹ For a constant one-form A_{D0}^{RR} , the five dimensional integral (1.2) is localized on the D0-brane world-line. If the proper time integral extends in the interval $\tau \in [0, L]$, we have $S_{\text{source}} = -i \frac{\theta}{L} \int d\tau$.

dynamics of 0-0 strings and 0-4 strings in the general case where we have k D0-branes bound to n_c D4-branes. The 0-0 sector is just the dimensional reduction down to one dimension of ten-dimensional SYM with $U(k)$ gauge group, corresponding to a system of k D0-branes, and has $N = 4$ supersymmetry in four-dimensional notation. The 0-4 sector breaks half of these unbroken 16 supercharges, leading to an action for the 0-4 strings with $N = 2$ supersymmetric couplings. To be more explicit, let us parametrize the relevant degrees of freedom in terms of a T-dual configuration of D3-branes inside D7-branes of type IIB string theory. We will take the D3-branes world-volume to lie in the (x^6, x^7, x^8, x^9) plane, with x^6 euclidean time, and the D7-branes to extend further in the (x^0, x^1, x^2, x^3) , all euclidean, directions. (These conventions for the axes follow from the conventions of ref. [2] where the world-volume of the D4-branes is along the $(x^0, x^1, x^2, x^3, x^6)$ directions, and the request that the euclidean-time world-line of the D0-branes is in the compactified Neumann direction of the D4-branes, that is along x^6 .) Using $N = 1$ superspace notation, the 0-0 sector contains a vector superfield built from the gauge field (A_6, A_7, A_8, A_9) on the D3-brane world-volume, and the “gluino” superpartners, and also three chiral superfields in the adjoint of the $U(k)$ gauge symmetry: W, Z, \tilde{Z} , whose bosonic components are the complex combinations $X^4 + iX^5, X^0 + iX^1, X^2 + iX^3$ respectively, and correspond to the transverse motion of the D3-brane. Upon dimensional reduction to the D0-brane world-line, we will gauge-fix A_6 to zero, and consider only configurations with $A_7 = 0$, so that the remaining components of the gauge field can be assembled into another chiral superfield $A_8 + iA_9 = X_8 + iX_9 = V$. The fermionic components of these superfields exhaust the $16k$ fermionic components of the ten dimensional $U(k)$ SYM theory. If we denote the fields $\Phi_{00} = (W, Z, \tilde{Z})$ collectively by $\Phi_i, i = 1, 2, 3$, the 0-0 action is defined by the following $N = 4$ superpotential:

$$\mathcal{W}_{0-0} = \frac{\sqrt{2}}{g_s} \frac{1}{3!} \epsilon_{ijk} \text{Tr } \Phi_i [\Phi_j, \Phi_k] . \quad (1.3)$$

The 0-4 sector contributes hypermultiplets in the fundamental and anti-fundamental representations of the $U(k)$ symmetry, with the D4-brane gauge indices $a, b, \dots = 1, \dots, n_c$ playing here the role of “flavour”. The 0-4 strings contribute a “D04-quark superfield” $\Phi_{04} = H = (\phi_h, \psi_h)$, and the orientation-reversed 4-0 sector gives the D40-anti-quark $\Phi_{40} = \tilde{H} = (\tilde{\phi}_h, \tilde{\psi}_h)$. By coupling these hypermultiplets to the previous 0-0 chiral superfields in the adjoint of $U(k)$, we break $N = 4$ supersymmetry to $N = 2$ and the $SO(6)$ global R-symmetry down to $SU(2)_R$. Indeed the D04-quark and D40-anti-quark fields

(H, \tilde{H}) couple in the superpotential to only one of the three 0-0 superfields. The 0-0 superfield which is singled out in this way is the W superfield according to its four-dimensional field theory interpretation. The corresponding superpotential reads

$$\mathcal{W}_{0-4} = \frac{\sqrt{2}}{g_s} (\tilde{H} W H - H W' \tilde{H}), \quad (1.4)$$

where we have introduced a mass term inherited from the superpotential in the D4-branes, i.e. the 4-4 strings, coupling the $SU(n_c)$ adjoint $W'_{44} = X'_4 + iX'_5$ giving the position of D4-branes. The $N = 2$ supersymmetry on the D0-brane world-line implies that $[W', W'^\dagger] = 0$, i.e. the W' field lies along the flat directions of the D4-brane theory, and can be taken as diagonal matrices by a $SU(n_c)$ rotation² $W'^b_a = w'_a \delta^b_a$.

The instanton moduli space is then obtained as a maximal Higgs phase of this theory on the D0-brane world-line, where we consider the most general vacuum expectation values of the hypermultiplets H, \tilde{H} and the adjoint fields Z, \tilde{Z} . In this paper we will be mainly concerned with properties of the single instanton sector, or equivalently, of the dilute gas limit of instantons. In this case the overlap between the instantons can be neglected and the moduli space admits a much simpler description. Indeed, well separated D0-branes are characterized by the complete diagonalization of all the adjoint 0-0 fields corresponding to the space-time positions, i.e. we consider Z, \tilde{Z} diagonal generic matrices, which spontaneously break $U(k)$ to the abelian subgroup $U(1)^k$. It is intuitively clear that this produces just k copies of the single instanton moduli space. In this subspace, the $4k$ position degrees of freedom z_i, \tilde{z}_i are completely decoupled from the rest of the variables. The hypermultiplets D -equations collapse to

$$\left(\phi_h \phi_h^\dagger - \tilde{\phi}_h^\dagger \tilde{\phi}_h \right)_m^m = 0, \quad (1.5)$$

and the F -equations, for diagonal W and W' fields:

$$\begin{aligned} (w_m - w'_a)(\phi_h)_a^m &= 0 \\ (w_m - w'_a)(\tilde{\phi}_h)_m^a &= 0 \\ (\phi_h)_a^m (\tilde{\phi}_h)_m^a &= 0 \end{aligned} \quad (1.6)$$

² The brane construction of the full gauge theory produces naively a $U(n_c)$ group. Only the $SU(n_c)$ subgroup is relevant to the instanton discussion, of course. In fact, on the Coulomb branch of the D4-branes world-volume, the extra $U(1)$ is frozen, since fluctuations of the trace of W' have infinite norm [16].

where $m = 1, \dots, k$ is an index labeling the D0-brane corresponding to each $U(1)$ factor, and a, b, \dots are the flavour D4-indices. The interesting branch of solutions is the one with maximal Higgsing of the hypermultiplets, with $w_m = w'_a = 0$, all m, a (all D0 and D4-branes on top of each other in the (x^4, x^5) plane). Subtracting the $2k$ F -equations and the k D -equations from the total $4kn_c$ real components in the scalars $\phi_h, \tilde{\phi}_h$, and further dividing by the $U(1)^k$ gauge symmetry, we find a total of $4kn_c - 2k - k - k = 4k(n_c - 1)$ real parameters. Adding the $4k$ degrees of freedom coming from the eigenvalues of Z and \tilde{Z} , representing the translation moduli in the physical dimensions, we end up with the correct $4kn_c$ dimensional bosonic moduli space of YM instantons.

The previous parametrization is appropriate for the dilute instanton limit, in which the non-abelian structure of $U(k)$ is not really probed. Indeed, when the instantons, or D0-branes “overlap”, the off-diagonal entries in the Z, \tilde{Z} fields become important and eqs. (1.5) and (1.6) are modified by terms which mix the adjoint superfields with the hypermultiplets. In any case, in the Higgs branch relevant to our discussion, Z and \tilde{Z} contribute $4k$ degrees of freedom and the hypermultiplets $4k(n_c - 1)$, describing as before the correct $4kn_c$ dimensional bosonic moduli space of YM instantons.

2. D-brane construction of exact instantons in SYM

In this section we adapt the previous construction to the more detailed realizations of SYM theories in four dimensions with different amounts of supersymmetry, starting with the maximal one, $N = 4$ SYM.

2.1. $N=4$ SYM

Since the world-volume theory of the D4-branes already has $N = 4$ $SU(n_c)$ SYM theory in five dimensions, we simply have to compactify the D4-D0 bound states to four dimensions, without any further breaking of supersymmetry. Consider wrapping both the D4 and D0-branes around a circle of length L_6 in the x_6 direction.³ On the relevant bosonic moduli space of real dimension $2d = 4kn_c$, the D0-branes world-line has a set of d free superfields (ξ_s, η_s) , in four-dimensional notation, i.e. we have an action

$$S_{D0} = kM_{D0} \int_0^{L_6} d\tau - ik\theta + \frac{1}{g_s} \int_0^{L_6} d\tau \sum_{s=1}^d \left(|\dot{\xi}_s|^2 + i\bar{\eta}_s \dot{\eta}_s \right) + \text{massive} . \quad (2.1)$$

³ Recall that this corresponds to the euclidean time of the D0-branes.

Each Weyl fermion η_s has two complex or four real components off-shell so that, when the one-instanton contribution to the path-integral is dominated by the classical static trajectory $\dot{\xi}_s = \dot{\eta}_s = 0$, we get a factor

$$Z_{\text{inst}} = \int \prod_s d\bar{\eta}_s d\eta_s d\mu(\xi_s) e^{-kM_{D0}L_6 + ik\theta} = 0^{4d} \text{Vol}(\mathcal{M}) e^{-\frac{8\pi^2 k}{g^2} + ik\theta} \quad (2.2)$$

where we defined $M_{D0}L_6 \equiv 8\pi^2/g^2$. In this expression, the bosonic measure $d\mu(\xi)$ for collective coordinates contains zero-mode Jacobians which are most easily derived in the field theoretical framework. The corresponding integral gives the formally divergent volume of moduli space, $\text{Vol}(\mathcal{M})$. The symbol 0^{4d} denotes the number of fermionic zero modes of the instanton, to be saturated by fermion sources. We see clearly that the number of zero modes corresponds to the total number of independent anti-commuting collective coordinates, which in turn coincide with the total number of off-shell fermion components on the D0-brane world-line.

The first important check of expression (2.2) is this number of zero modes $4d = 8kn_c$ which is indeed the right one for $SU(n_c)$ SYM in the k -instanton sector. In field theory, each gluino system gives $2kn_c$ zero modes, and we have four independent gluino systems in $N = 4$ SYM. In the D0-D4 branes construction, the $8k$ zero modes associated to the Z, \tilde{Z} superfields are interpreted in the four dimensional SYM theory as supersymmetry zero modes. On the other hand, $8k$ out of the $8k(n_c - 1)$ zero modes associated to the hypermultiplets H, \tilde{H} are interpreted in space time as superconformal zero modes, while the remaining $8k(n_c - 2)$ zero modes do not have an obvious symmetry interpretation, since they arise as 't Hooft zero modes associated to doublets with respect to the $SU(2)$ subgroup of the gauge group where the instanton sits.

The second check of (2.2) is given by the fit of the Yang-Mills coupling to the mass of the D0-brane, $M_{D0}L_6 \equiv 8\pi^2/g^2$, needed to reproduce the standard instanton factor. Dimensional reduction from the D4-brane Born-Infeld action leads to

$$\tau_4 \int_0^{L_6} d\tau \text{Tr} \sqrt{\det(1 + 2\pi\alpha' F)} \rightarrow -\frac{\tau_4}{4} L_6 (2\pi\alpha')^2 \text{Tr} F^2 \equiv -\frac{1}{4g_{YM}^2} \text{Tr} F^2. \quad (2.3)$$

Using $\tau_p^{-1} = g_s \sqrt{\alpha'} (2\pi\sqrt{\alpha'})^p$ and $M_{D0} = \tau_0$ we find agreement with (2.2), i.e. $g^2 = g_{YM}^2$. This is an important point, since it implies that the length of the D0-branes world-line to be considered is really L_6 , a statement which will become non-trivial in the next section, when considering the case of $N = 2$ and $N = 1$ SYM theories.

2.2. $N=2$ and $N=1$ SYM

More interesting is the D-brane construction of instantons with reduced supersymmetry in the four physical dimensions. Using the constructions of refs. [1] and [2], we compactify the five-dimensional theory to four dimensions on the D4-brane world-volume by means of a Kaluza-Klein reduction on a segment of length L_6 in the x^6 direction, bounded by NS five-branes with world-volume in the $(x^0, x^1, x^2, x^3, x^4, x^5)$ directions. By a suitable complex rotation of the (x^4, x^5) plane into the (x^8, x^9) plane of one of the NS-branes [4], one breaks further the $N = 2$ supersymmetry to $N = 1$. In such constructions, the role of the NS-branes is simply to project out certain degrees of freedom, and the appropriate supersymmetries. They do not add any extra degrees of freedom, unless special classes of phase transitions are considered. In particular, the fermions propagating in the world-volume of the NS-branes are set to their vacuum values. Among them, we have the six-dimensional Goldstone fermions associated to the supercharges broken by the NS-brane. Therefore, in constructing the relevant Yang-Mills instantons using D0-branes, it is natural to project out those fermion zero modes which overlap with the above-mentioned six-dimensional Goldstone fermions. This makes sense because we consider constant fermion configurations, $\dot{\eta} = 0$, on the D0-brane world-line, which extends between the NS branes, intersecting them at the end-points.

Recalling (2.3), we see that the D0-brane world-line has length L_6 in order to fit the bare Yang-Mills instanton action, and therefore it extends only once between the NS-branes. This world-line touching the NS-branes at the end-points must be considered as topologically stable, unlike the case of free D0-branes or the $N = 4$ setting without NS-branes, where a finite D0-action contribution requires wrapping the world-line around some non-contractible circle in the target space. This distinction between free D0-branes and bound D0-branes will reappear in our comments on the relation with the M-Theory lifting in the last section.

On the other hand, it is important that no extra constraints are imposed by the NS-branes on the bosonic collective coordinates ξ_s , whose measure is completely determined in (2.2) in the $N = 4$ theory, up to Jacobian factors which are related to zero modes. In the absence of a detailed understanding of the microscopic couplings between D-branes and NS-branes, we have to rely on consistency checks with the field theory picture and geometric arguments. For example, the projection of certain bosonic degrees of freedom on the world-volume of the D4-branes follows from the global geometry once we assume the

rigidity of the NS-branes. Alternatively, the bosonic projections follow from the fermionic ones by supersymmetry. In our case, the brane geometry projects out the scalars in the 0-0 sector, except those representing the motion of the D0-brane in the physical space-time (x^0, x^1, x^2, x^3) , but there is no geometric constraint on the hypermultiplet degrees of freedom of the 0-4 and 4-0 sectors, and therefore no obvious geometric constraint imposed by the NS-branes on the associated collective coordinates ξ_s . The intersection of the D0-brane world-line with the NS branes occurs at isolated points in space-time and, as a result, there is no hamiltonian representation of bosonic and fermionic degrees of freedom on the instanton “world-volume”, and therefore no statement of equality between fermionic and bosonic “states”, as it would be required by supersymmetry in non-vanishing dimensions. Exactly the same mechanism is at work in the standard instanton superfield formalism [17,18,19].

In order to see how the fermion projections work, consider the case of two NS-branes and a set of n_c D4-branes and k D0-brane world-lines suspended between them in the x^6 direction. Denote by NS' the second NS-brane, which is parallel to the first NS-brane but translated in the x^6 direction by L_6 in the $N = 2$ configuration, and rotated into the (x^8, x^9) directions for the $N = 1$ configurations. We have seen that $8k$ of the fermionic collective coordinates in the bulk come from the fermionic superpartners of the superfields Z, \tilde{Z} , representing translations in the space-time (x^0, x^1, x^2, x^3) directions. In the D-brane construction, they arise in the 0-0 sector, and can be interpreted as part of the Goldstone fermions of the broken supersymmetries by the D0-brane world-line. An isolated D0-brane breaks 16 of the total 32 supercharges of the type-IIA theory, and accordingly we find 16 Goldstone fermions on the world-line. These fermions were assembled in the first section into four four-dimensional Weyl fermions: $\psi_z, \psi_{\bar{z}}, \psi_w, \psi_v$. As we have stressed in different occasions, only half of them, namely $\psi_z, \psi_{\bar{z}}$ survive the Higgs mechanism on the branch of D0-D4 branes bound states.

In general, in the present type-IIA context, the projector over the unbroken supersymmetries on a D-brane world-volume is best represented in M-Theory language as

$$P_p = \frac{1}{2}(1 + i\Gamma(\Sigma_{p+1})) \quad (2.4)$$

where $\Gamma(\Sigma_{p+1})$ is the product of Dirac matrices in the world-volume directions, with the understanding that p -branes whose M-Theory description involves a wrapped or boosted M-brane in the eleventh direction include a factor of Γ_{11} [20]. The factor of i in (2.4)

stands for the euclidean rotation of Γ_0 , as appropriate for the instanton discussion. So, in terms of a 32-dimensional spinor Ψ_{32} , the 16 Goldstone fermions carried by each D0-brane are given by $(1 - P_{D0})\Psi_{32}$. Now, according to the prescription above, we should keep only those zero modes with no overlap with the Goldstone fermions on the NS-brane world-volume. The remaining zero modes from the 0-0 sector of a general D0-brane are then

$$\eta_{z,\tilde{z},v,w} = (1 - P_{D0})P_{NS}P_{NS'}\Psi_{32} . \quad (2.5)$$

Using the explicit form of the projectors, one easily sees that each NS-brane divides by half the number of zero modes, so that we have 8 zero modes in the $N = 2$ configuration, and 4 zero modes in the $N = 1$ configuration.

The previous considerations apply to the situation of a free D0-brane. In the case of a D0-D4 branes bound state we have two modifications. First, as stated before, on the Higgs branch of bound states the fermions ψ_v, ψ_w are lifted, and the number of zero modes (2.5) coming from the 0-0 sector is again divided by half. Second, we have a new set of degrees of freedom, namely the $\psi_h, \tilde{\psi}_h$ from the 0-4 and 4-0 sectors. Those are constructed from the zero modes of the world-sheet fermions ψ_0^μ for $\mu = 4, 5, 6, 7, 8, 9$ (i.e. in the NN + DD directions of the D0-D4 brane system), in the Ramond sector. Indeed, there are $\frac{1}{2}2^{6/2} = 4$ field components (off shell), after GSO projection, in the 0-4 sector, that is a Weyl fermion. In terms of the complex combinations $\Gamma_u = \frac{1}{2}(\Gamma_6 + i\Gamma_7)$, $\Gamma_w = \frac{1}{2}(\Gamma_4 + i\Gamma_5)$, $\Gamma_v = \frac{1}{2}(\Gamma_8 + i\Gamma_9)$, the chirality operator reads

$$\Gamma_{11} = i\Gamma_0\Gamma_1 \cdots \Gamma_9 = -(1 - 2N_z)(1 - 2N_{\tilde{z}})(1 - 2N_u)(1 - 2N_v)(1 - 2N_w), \quad (2.6)$$

with $N_v \equiv \Gamma_v^\dagger \Gamma_v$, etc., the fermion occupation numbers taking values 0 or 1. Acting on the $2^{6/2} + 2^{6/2}$ states in 0-4 and 4-0 sectors, built as polynomials $P(\Gamma_u^\dagger, \Gamma_v^\dagger, \Gamma_w^\dagger)|0\rangle$, the GSO projection reduces to $-(1 - 2N_u)(1 - 2N_v)(1 - 2N_w) = +1$. On the same set, the projection imposed by the NS brane is $+1 = i\Gamma_{NS} = i\Gamma_0\Gamma_1 \cdots \Gamma_4\Gamma_5 = -(1 - 2N_w)$. So, on this subspace the NS projector is $P_{NS} = N_w$, and the analogous projector for the rotated NS'-brane is $P_{NS'} = N_v$.

We see that, in all cases, the effect of each NS-brane is to divide by 2 all the bulk zero-modes democratically. This amounts to a total factor of 1/4 when the NS' brane is a rotation of the NS-brane into the (x^8, x^9) plane. So, if N denotes the amount of space-time supersymmetry of the brane configuration, the resulting instanton leaves $2N$ supercharges

unbroken, out of the total $4N$ which are linearly realized in the vacuum of the space-time theory. The total number of fermionic zero-modes is given by:

$$\mathcal{N}_{f.z.m.} = \frac{N}{4} \cdot 4d = \frac{N}{4} \cdot 8k + \frac{N}{4} \cdot 8k(n_c - 1) = 2Nkn_c, \quad (2.7)$$

the correct number, where we have already subtracted the massive degrees of freedom in the Higgs branch which defines the instanton moduli space, and we have split the zero modes between those corresponding to the $2N$ broken supercharges, and those of a fixed instanton, coming from the hypermultiplet degrees of freedom.

Notice that in the preceding sections, it was convenient to adopt the notation in terms of Weyl fermions with respect to the $SO(1, 3)$ of the (x^6, x^7, x^8, x^9) plane. However, the presence of the NS or NS'-branes breaks this group, and the fermion components left after projection need not fulfill representations of such group.

3. Constrained instantons and the Coulomb branch

Up to now we have considered the case of instantons in super Yang-Mills theory and showed that it can be described by a system of k D0-branes bound to n_c D4-branes. In particular this means that the D0-branes and the D4-branes are all located at the same point in the $(x^4, x^5, x^7, x^8, x^9)$ plane. In this section we will show that the geometrical operation of moving apart the D4-branes corresponds, as intuition would tell, to describing constrained instantons [21,22,23]. We postpone the introduction of space-time matter, i.e. D6-branes, and its complications to the next section. Thus in this section we will study the Coulomb phase of $N = 2$ SYM models.

Let us start by considering what happens in moving apart the D0-branes. For example, separating one D0-brane from the others in the (x^4, x^5) directions, just means that one is considering a bundle of $(k - 1)$ instantons. In other words, only when the D0-branes are bound to the D4-branes (in the $(x^4, x^5, x^7, x^8, x^9)$ plane) they can describe exact instantons. It obviously follows that, in moving apart a D4-brane from the others, we will leave the D0-branes bound to the $(n_c - 1)$ remaining D4-branes, and that we can only speak of exact instantons on the corresponding $SU(n_c - 1)$ subgroup. We can, however, still consider

approximate (“constrained”) instantons of the full $SU(n_c)$ group, by moving a little apart, one by one, the D4-branes while keeping the D0-branes bound to at least two D4-branes.⁴

In the following we shall concentrate on the single instanton sector, i.e. only one D0-brane, and return at the end of the section to the more involved case of multi-instanton interactions. For a single D0-brane only the relative position with respect to the D4-branes matters, and therefore we may fix the D0-brane at $w = 0$, and consider the effect of turning on the W' field, whose diagonal configurations correspond to well defined D4-brane positions.

The relevant coupling in the D0-brane world-line theory which is sensitive to the D4-branes motion is (see eq. (1.4))

$$\mathcal{L}_{04-44-40} = \frac{\sqrt{2}}{g_s} \int d^2\theta \Phi_{04} \Phi_{44} \Phi_{40} + \text{h.c.} \quad (3.1)$$

This coupling is simply a mass term (the mass matrix given by $\phi_{44} \sim W'$ which describes the position of the D4-branes in the (x^4, x^5) plane) for the instanton moduli (in the notation of (1.4) $\Phi_{04} \sim \tilde{H}$ and $\Phi_{40} \sim H$). When considered in the Coulomb branch of the $N = 2$ configurations, it yields the familiar action of a constrained instanton:

$$S_{\text{constr}} = \frac{1}{g^2} (8\pi^2 + \rho^2 v^2) , \quad (3.2)$$

with $v \sim \langle \phi_{44} \rangle$ and $\rho^2 \sim \phi_{04} \phi_{40}$. At the same time, it lifts the fermion zero modes through the pairings $v \psi_{04} \psi_{40}$. For the fermions, the lifting of zero modes occurs homogeneously in the bulk of the D0-brane world-line, and therefore it is obviously compatible with the projections imposed by the NS-branes at the boundaries.

In general, an homogeneous lifting pattern of this sort, appropriate to describe the Coulomb phase of $N = 2$ models, gives a mass of order v to d_c complex bosonic collective coordinates, and similarly, it lifts $4d_c$ real fermionic coordinates in the bulk, i.e. it induces a term in the bulk action of the form

$$\delta \mathcal{L} \sim v^2 \sum_{s=1}^{d_c} |\xi_s|^2 + v \sum_{s=1}^{d_c} \bar{\eta}_s \eta_s , \quad (3.3)$$

⁴ Indeed in field theory we need at least $SU(2)$ to be able to build YM instantons. Analogously, the D-brane construction collapses if $n_c < 2$, because there is no $N = 2$ Higgs branch for less than two flavours.

where we have rescaled the bare Yang-Mills coupling $g^2 \sim g_s/L_6$ into the collective coordinates ξ_s and η_s . Taking into account the projection at the boundaries of the world-line due to the NS and NS'-branes, we have a partition function of the form

$$Z_{\text{inst}} \sim 0^{N(d-d_c)} \text{Vol}_c(\mathcal{M}) v^{\frac{Nd_c}{2}} e^{-\frac{8\pi^2}{g^2} + i\theta} \quad (3.4)$$

where $\text{Vol}_c(\mathcal{M})$ represents the bosonic “volume” of the moduli space, regulated with the gaussian term $\exp(-v^2|\xi|^2)$, and the term $v^{Nd_c/2}$ comes from the fermionic integrals.⁵ The complete power of v in the full instanton measure depends on the result of the bosonic integrals, of course. In general, we must view the couplings like (3.1) as small perturbations over the instanton moduli space described in the introduction. This amounts to the requirement that the dimensionless effective expansion parameter ρv be small.

The important remark to make is that all zero modes due to the conformal symmetry and embedding angles, namely, all zero modes encoded in Φ_{04}, Φ_{40} , can be lifted in this way on the Coulomb branch of the $N = 2$ theory, by switching on Φ_{44} (i.e. splitting the D4-branes in the (x^4, x^5) plane). This amounts to lift generically $2d_c = 4(n_c - 1)$ bosonic and $4(n_c - 1)$ fermionic zero modes. The remaining zero modes, 4 bosonic and 4 fermionic, correspond to the superfields from the 0-0 sector Z and \tilde{Z} , which remain uncoupled and therefore are not lifted. Since these collective coordinates are associated to the translations in the physical four dimensions, these fermion zero modes are naturally interpreted as due to the broken supersymmetries of the instanton, understood as a classical solution in the physical four-dimensional theory on the fourbranes.

We now briefly sketch the situation in the case of multi-instanton configurations. In the limit of dilute instantons, the lifting pattern of the zero-modes is just given by k copies of the single instanton case. Instead, beyond the dilute approximation, new terms appear in the effective lagrangian, due to the non-vanishing of the off-diagonal entries of the Z, \tilde{Z} fields. For example, in the parametrization given in ref. [24] of the Higgs branch of the effective theory of the D0-branes, one can exhibit particular configurations with diagonal Z , but non-trivial off-diagonal entries of the \tilde{Z} field, given by

$$\tilde{Z}_n^m \sim -\frac{(\phi_h)_a^m (\tilde{\phi}_h)_n^a}{z_m - z_n} \quad (3.5)$$

⁵ Recall that N is the number of four-dimensional supersymmetries in the space-time configuration, and $d = 2n_c$ is the complex dimension of the bosonic instanton moduli space.

as a function of the eigenvalues z_n of the Z fields, and the hypermultiplets' moduli. Now the term $||[W, \tilde{Z}]|^2$ of the effective potential vanishes unless at least one of the two fields has non-vanishing off-diagonal entries. Thus in presence of the non-trivial off-diagonal entries of the \tilde{Z} field given by eq. (3.5), and turning on a diagonal W field, representing definite separations of the D0-branes in the (x^4, x^5) plane, we find positive energy contributions to the effective potential of the form

$$\frac{|w_m - w_n|^2}{|z_m - z_n|^2} (\phi_h)_a^m (\tilde{\phi}_h)_n^a (\phi_h^*)_b^n (\tilde{\phi}_h^*)_m^b \quad (3.6)$$

which describe a runaway potential for the eigenvalues z_n , and an analogous lifting term for the superpartners. Notice that this potential depends only on the relative separations $z_m - z_n$, so that the overall center of mass of all D0-branes is not lifted by these interactions, and furthermore (3.6) vanishes in the limit of instantons at very large distances, i.e. the dilute limit. This is in agreement with the known results in field theory (see for example ref. [19]) where it has been shown that on the Coulomb branch there remain only 4 bosonic and 4 fermionic zero modes, associated with the center of mass position of the instantons and with the broken supersymmetries, respectively.

4. Space-time flavour and the Higgs branch

In the D-brane formulation of super-QCD (SQCD), the matter (space-time) flavour is described by n_f D6-branes with world-volume in the $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$ directions. We know from field theory that the main effect of the presence of matter on the instantons is the appearance of new fermionic zero modes, as well as new patterns of zero mode lifting when turning on the Higgs branch of the field theory. In this section, we will start by describing the flavour zero modes of the instanton in terms of the local dynamics of the D0-branes and the D6-branes. An important aspect of this dynamics, and a source of many subtleties, is the non-standard geometrical arrangement of the D0-D6 branes system. In our construction, the D0-branes are extended in euclidean time in the x^6 direction, while the D6-branes are completely localized on this axis. Therefore, D0 and D6-branes do not share any propagating dimension, and are purely instantonic relative to each other. This changes the standard statement that the D0-D6 branes system is not supersymmetric. Here the relevant parameter $\nu = d_{ND} + d_{DN}$, the sum of ND and DN directions in the

boundary conditions of 0-6 and 6-0 strings, takes the value $\nu = 8$ and we have some unbroken supersymmetry.

The local spectrum on the 0-6 and 6-0 strings is similar to the standard $\nu = 8$ spectrum at the intersection between a D1-brane and a D9-brane, as explained for example in [25]. We can transform the D0-D6 brane system at hand into a D1-D9 brane pair by means of a T-duality transformation along the (x^4, x^5, x^6) directions and considering x^4 as the (euclidean) time. In the NS sector the Casimir energy of 0-6 strings is $-1/2 + \nu/8 = +1/2$ and therefore there are no bosons at all made from 0-6 strings. In the Ramond sector the worldsheet fermions ψ^μ in the ND or DN directions have half-integer moding, and so the corresponding vacuum is non-degenerate. We are left with the two fermionic zero modes in the two DD (or NN under T-duality) directions. This leads to two states, one of which is projected out by the GSO projection. The physical state condition, or Dirac equation $G_0 = p_\mu \psi^\mu = 0$, which imposes a holomorphic constraint on the D1-brane excitations of the D1-D9 brane system, here degenerates due to the instantonic nature of the D0-D6 intersection. In our case, where the D0 and the D6-branes do not share any propagating dimensions, we have no physical state condition. In all, adding the 0-6 and 6-0 sectors, we would have a total of $2kn_f$ fermionic collective coordinates, the correct number we expect from field theory considerations. Due to the absence of a physical state condition, we have to think of these localized states as effectively off-shell, and therefore, we must have $2kn_f$ holomorphic Grassmannian integrals in the instanton measure, taking the form

$$d\mu_{\text{flavour}} = \prod_{j=1}^{n_f} d\chi_j^* d\chi_j = d\chi_{60} d\chi_{06}. \quad (4.1)$$

The two states at each D0-D6 branes intersection can be assembled into a complex field $\chi = \chi_{06}$, with the orientation reversed component χ_{60} representing the conjugated field χ^* . In this way we may realize the $U(1)$ gauge symmetry of the D6-brane as a global symmetry on the D0-D6 branes intersection.⁶

Now we can consider the full system adding the D4-branes and the NS-branes. By adding the D4-branes the space-time theory becomes SQCD with n_f flavours and we expect that in the world-line theory of the D0-branes there will appear induced couplings due to

⁶ Notice that, in this picture, the full $U(n_f)$ global flavour symmetry is hard to visualize explicitly, since the D6-branes sit at different “instants” of time from the point of view of the D0-brane world-line.

the 4-4 and 4-6 strings. As before the NS and NS'-branes give us $N = 2$ or $N = 1$ SQCD. The only new feature is the effect of the projections induced by the NS and NS'-branes on the 0-6 strings. The claim is that there is no new projection and thus the number of zero modes due to the 0-6 strings does not change with respect to the analysis in the previous paragraph. This can be understood as follows. Recall that the D6-branes are completely instantonic with respect to the zero branes and that in the x^6 space they are generically located between the NS and NS'-branes. Thus 0-6 strings are localized at the intersection of the D6-branes with the D0-branes world-line. For this reason they do not feel the presence of the NS and NS'-branes (unless of course the D6-brane is on top of the NS or NS'-brane, but this is a non generic configuration that we will not consider). Notice that the situation is quite different for the 0-4 strings, since the D0-branes are bound to the D4-branes in the x^6 direction, the 0-4 strings are not localized and thus feel the projections due to the NS and NS'-branes at the boundaries. The same of course happens for the 0-0 strings. The independence of the 0-6, 6-0 modes upon the boundary projection is in beautiful correspondence with the independence of the quark instanton sector upon the amount of supersymmetry in SQCD.

After our description of the exact instanton in the SQCD at the origin of moduli space, we can discuss the patterns of zero-mode lifting when exploring the moduli space of the space-time theory. In the brane picture this corresponds to motions of the background branes, and the corresponding liftings are understood in terms of induced couplings in the D0-branes world-volume. As before, we will concentrate mainly on the single instanton sector (or the dilute instanton case).

First of all, there is the trivial case of adding a mass to the quarks in space-time, or moving the D6-branes away from the D4-branes in the (x^4, x^5) plane. This can be described on the D0-branes world-line as a local coupling of the χ_{06}, χ_{60} variables to the scalar ϕ_{66} taking a non-vanishing expectation value under relative D4-D6 branes displacement:

$$\mathcal{L}_{06-66-60} \sim \chi_{06} \phi_{66} \chi_{60} \delta(\tau - \tau_{D6}). \quad (4.2)$$

As expected, this coupling lifts the associated flavour zero modes of the instanton, leaving all other fermionic and bosonic collective coordinates intact.

A more complicated pattern appears when exploring the Higgs branch of the space-time theory, by switching on the expectation values of the squark fields $\langle \phi_{46} \rangle, \langle \phi_{64} \rangle \neq 0$. In the brane picture, D4-branes corresponding to the Higgsed color indices are broken on

the D6-branes and lifted into the (x^7, x^8, x^9) directions in the $N = 2$ configurations, or into the (x^8, x^9) plane in the case of the $N = 1$ setting. Now, the corresponding instanton coordinates, built from some components of the Φ_{04}, Φ_{40} superfields, get masses due to the finite stretching of the 0-4 and 4-0 strings. This stretching occurs in the bulk of the D0-brane world-line, along the lifted portion of the D4-brane, and not only at the location of the D6-branes. Therefore, the bulk mass of the lifted Φ_{04}, Φ_{40} fields is due to a coupling which can be written with no reference to the D6-branes. It simply corresponds to an expectation value of the Φ_{44} fields in the appropriate directions. Now, in the notation adopted in section 1, in order to write the D0-brane world-line supersymmetry in four-dimensional superspace notation, we represented the D0-D4 fields as T-duals of a D3-D7 brane system whose intersection lied in the “space-time” (x^6, x^7, x^8, x^9) . In this notation, a motion of the D4-brane in the (x^8, x^9) plane is represented as a background gauge field $V'_{44} = A'_8 + iA'_9$ on the D7-brane world-volume. Going back to the D0-brane world-volume via dimensional reduction, we have to consider a non-zero expectation value of the “gauge field” V'_{44} , coupling to the “flavour indices” of Φ_{44} . So, from this point of view, the appropriate masses come now from D-terms rather than holomorphic couplings on the D0-branes world-line. Taking $\langle \phi_{v'} \rangle \sim \langle \phi_{46} \rangle \sim \langle \phi_{64} \rangle \sim v$, we have induced masses of the form (3.3) for the bosonic and fermionic components of the Φ_{04}, Φ_{40} superfields, containing the collective coordinates of a fixed instanton.

On the other hand, the $2n_f$ flavour collective coordinates found above, χ_{06}, χ_{60} , are localized at each D0-D6 intersection, and must be lifted by means of a local coupling involving all three types of branes touching locally at that point, including the D4-branes. The reason is that, as we know from the field theory analysis, the flavour zero modes are lifted by pairing them with the rest of the fermionic zero modes of the instanton. In the D-brane setting, we must pair up the flavour collective coordinates with the fermion zero modes from the 0-4 and 4-0 sectors. Now, the most general coupling respecting the orientation reversal symmetry of the string theory, hermiticity, and the $U(1)$ global symmetry of the D6-branes, is given by

$$\mathcal{L}_{06-64-40} \sim \left[\chi_{06} \left(\phi_{64} \psi_{40} + \phi_{46}^* \psi_{04}^* \right) + \chi_{60} \left(\psi_{04} \phi_{46} + \psi_{40}^* \phi_{64}^* \right) \right] \delta(\tau - \tau_{D6}) , \quad (4.3)$$

where we have used the fact that the complex conjugate of a 0-4 field, ψ_{04}^* , transforms like a 4-0 field.

In order to exhibit in more detail the pattern of pairings, it is convenient to consider in turn the two cases with $N = 2$ and $N = 1$ space-time supersymmetry. In the first case, the NS projection leaves $4n_c$ holomorphic fermionic collective coordinates of the 0-4 and 4-0 strings, prior to any Higgsing on the instanton moduli. We can denote them as $\psi_a, \tilde{\psi}_a^*$, transforming as ψ_{04} , and $\tilde{\psi}_a, \psi_a^*$, transforming as ψ_{40} . The part of the $N = 2$ Higgs branch accessible to the D-brane representation is the set of non-baryonic Higgs branches, which, according to ref. [26], can be parametrized, up to $U(n_c) \times U(n_f)$ transformations, by

$$(\phi_{64})_j^a = v_j \delta_j^a, \quad (\phi_{46})_a^j = v_{j-[n_f/2]} \delta_j^{a+[n_f/2]} \quad (4.4)$$

where $a = 1, \dots, n_c$, $j = 1, \dots, n_f$ and at most $r \leq [n_f/2]$ of the v_j are non zero. Then, the general coupling (4.3) takes the form

$$\mathcal{L}_{\tau=\tau_{D6}} \sim \sum_{j=1}^r v_j \left(\chi_j \tilde{\psi}_j + \chi_j^* \tilde{\psi}_j^* \right) + \sum_{j=[n_f/2]+1}^{r+[n_f/2]} v_{j-[n_f/2]} \left(\chi_j \psi_{j-[n_f/2]}^* + \chi_j^* \psi_{j-[n_f/2]} \right) \quad (4.5)$$

and we see clearly the geometric pattern of lifting characteristic of the $N = 2$ configurations: two D6-branes are necessary to lift one D4-brane, and also the zero mode lifting pattern is two to one, in the sense that two fermion components associated to the same D4-brane pair up with two flavour components associated to different D6-branes, those on which the D4-brane is sliding.

In the case of the $N = 1$ configuration, the NS-NS' projection leaves only $2n_c$ fermionic components of the 0-4 and 4-0 strings, which we can denote by ψ_a, ψ_a^* . Also, the Higgs constraints are different, and the Higgs phase is parametrized by $\phi_{46} = \phi_{64} = \text{diag}(v)$. Therefore, the general coupling (4.3) reduces now to

$$\mathcal{L}_{\tau=\tau_{D6}} \sim \sum_{j=1}^r v_j \left(\chi_j \psi_j + \chi_j^* \psi_j^* \right). \quad (4.6)$$

Now the pairing is one to one, as only one D6-brane is needed to lift each D4-brane.

The couplings (4.5) and (4.6) must be considered together with the induced masses in the bulk for the ψ_{04}, ψ_{40} fields, coming from the D-term couplings to the V' field. In addition, we have the masses induced by the Higgs effect on the instanton moduli space, i.e. the equations (1.5) and (1.6), which must be regarded as dominant, according to the prescription stated in section 3, that all couplings must be considered as perturbations of

the instanton moduli space, i.e. $\rho v \ll 1$. Thus, we have a complicated pattern of mixings between the fermionic zero modes on the Higgs branch.

In particular, in the case of $N = 2$ supersymmetry and $n_f = 2n_c - 2$ flavours on the non-baryonic branch, one can lift all flavour zero modes and $4(n_c - 1)$ fermionic zero modes of the fixed instanton. Only the 4 fermionic zero modes associated to the Z and \tilde{Z} fields remain, since these fields are spectators and are not lifted on all the branches of the model. This of course agrees with the field theory results.

Similarly, in the case of $N = 1$ supersymmetry with $n_f = n_c - 1$ flavours, on the Higgs branch one can lift all flavour zero modes and $2(n_c - 1)$ fermionic zero modes of the fixed instanton. The 2 remaining zero modes coming from the Z and \tilde{Z} fields, are those related to the appearance of the Affleck-Dine-Seiberg super-potential [27].

Thus in the case of $N = 1$ supersymmetry with $n_f = (n_c - 1)$ D6-branes, the presence of a D0-brane bound to the D4-branes gives rise, on the Higgs branch, to an effective repulsive interaction between the D4-branes. This effect is a particular case of the “quantum rules” proposed in ref. [8] for even more general configurations with $N = 1$ supersymmetry and an arbitrary number of D4-branes and D6-branes. The D0-branes, which act as the field theory instantons in these models, give rise to this “quantum rule” only in the $n_f = n_c - 1$ case. In fact, a more complete motivation of such quantum rules in terms of instantons requires studying the system compactified on a circle, and exploiting T-duality [3,8]. Under a T-duality transformation, D0-branes are converted into D1-branes of type-IIB string theory, and the associated instanton configurations look like dimensionally reduced monopoles of the theory with an additional adjoint scalar. The superpotentials on the Coulomb branch of this scalar are saturated by three-dimensional instantons for any number of flavours. This “fractionalization” property of instantons upon compactification is a well known phenomenon [28,29,30].

5. Discussion

In this paper we have studied the microscopic model of instantons in $SU(n_c)$ SYM theories as D0-branes bound to D4-branes in the Type-IIA D-brane description of the space-time theory. In particular, we have traced the phenomenon of zero-mode lifting of

constrained (approximate) instantons, to certain interactions in the local quantum mechanics of the D0-branes world-line. In this way we provide a number of non-trivial checks on the D-brane construction of Yang–Mills instantons, as presented in [12,13].

As Witten has first shown in ref. [16], and further discussed in refs. [31,32,33,34], these configurations of NS, D4 and D6-branes can be lifted to M-theory where one obtains the exact solution of the theory through the Seiberg-Witten curves which are nothing else than the algebraic curves defining the geometry of the branes in the internal, i.e. $(x^4, x^5, x^6, x^7, x^8, x^9, x^{10})$, space. Of course, the solution, being exact, contains all the contributions from the field theory instantons even if in the Type-IIA model only NS, D4 and D6-branes appear. Indeed, in lifting the Type-IIA configuration to M-theory, the NS-branes become the M-theory fivebrane, the D4-branes are also M-theory fibranes this time with one world-volume direction along the x^{10} axis. The D6-branes in M-theory are “Kaluza-Klein monopoles” or a multi-Taub-NUT space and they appear as a background internal space on which the M-theory fivebranes propagate. Thus the NS-branes and the D4-branes become in M-theory a single fivebrane whose algebraic equation is the Seiberg-Witten curve of the model. From M-theory point of view, the intersection between NS-branes and D4-branes is not “singular” anymore, and its resolution is described by the Seiberg-Witten curve.

Although our discussion applies strictly to the regime of weak type-IIA coupling, it is interesting to ask what happens of the D0-branes we have been discussing in this paper, in the lifting to M-theory. A free D0-brane of the type-IIA theory becomes a graviton Kaluza-Klein state carrying momentum along the circle in the eleventh direction, and electrically charged with respect to the $U(1)$ gauge field corresponding to the rotations around this circle. As discussed in refs. [35,36], the physics of such free D0-branes seems to have nothing to do with the field theory SYM models we are interested in. Whereas in the M-theory picture it could be not so simple to decouple these modes, in the Type-IIA construction we can just declare that we do not consider free D0-branes. In this way, we are making an explicit distinction between the free D0-branes, and the D0-D4 branes bound states. As stated in section 2, this distinction is natural from the point of view of the type-IIA configuration, since there is no non-contractible circle where we could wrap a free D0-brane world-line to yield a semiclassical instanton contribution. On the other hand, the world-line of the bound D0-brane is just the euclidean world-line of the D4-brane with which it is solidary. In other words, once the D0-branes are bound to the D4-branes, they must be considered as part of the dynamical data on the D4-branes world-volume.

Thus, the only D0-branes of interest are those which are bound to at least two D4-branes and are not allowed to become free. In the internal space, the D0-branes just completely adhere to the D4-branes since they differ only in the space-time directions where the D0-branes are at a point. In lifting to M-theory, only the internal space gets modified and from this point of view the bound D0-branes just completely merge in the D4-branes and they become part of the resulting fivebrane. Actually one could see this from the opposite point of view: in extending the D4-branes to become an M-theory fivebrane wrapped around the eleventh circle, one is effectively dressing them up with bound D0-branes.

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